PROGRAM: SUMMERSCHOOL 2009

Abstract. The aim of this summer school is to understand and explore some of the modern techniques used in the study of coherent sheaves over weighted projective lines (in the sense of Geigle and Lenzing). In particular we will cover the associated Lie algebras, which are loop algebras of Kac-Moody Lie algebras, and a construction due to Peng and Xiao of Lie algebras from certain triangulated categories using Hall algebra methods. At the end we will combine these ideas and cover the proof by W. Crawley-Boevey of an analogue of Kac's Theorem for weighted projective lines. In addition, there will a couple of overview talks shedding light on further developments and applications of certain concepts.
Each participant should contribute a talk and may indicate which topic they would care to talk about. The final schedule, however, will be arranged by the organizers and communicated to all participants in due course.

If you have any questions about the program please feel free to contact Marcel Wiedemann (marcel.wiedemann@math.upb.de).

1. **Coherent sheaves on weighted projective lines. (3 x 60 min)**
   (a) graded sheaf theory, Serre's Theorem
   (b) properties of the category $\operatorname{coh}\mathbb{X}$ (hereditary, abelian, finite dimensional homomorphism and extension spaces, Serre duality)
   (c) natural tilting configuration, canonical algebras
   (d) description of the subcategory of torsion sheaves supported at a given point; connection with nilpotent representations of cyclic quivers
   (e) torsion free sheaves, parabolic bundles on $\mathbb{P}^1$ and squids
   (f) Grothendieck group, Kac's Theorem
   References: [2], [3], [6], [13], [20], [21, Section 4.4]
2. **Kac-Moody algebras and loop algebras (3 x 60 min).**
   (a) introduction to Kac-Moody algebras (definition, bilinear form, quotient, triangular decomposition, generators of the radical, root system [esp. finite and affine type])
   (b) classification of f.d. simple Lie algebras via Dynkin diagrams
   (c) construction of affine Kac-Moody algebras as loop algebras from f.d. simple Lie algebras (universal central extension)
   (d) definition of loop algebras (input finite type gives corresponding affine type)
   (e) input affine type gives description in terms of generators and relations of the universal central extension of $\mathfrak{g}\otimes\mathbb{C}[t_1, t_1^{-1}, t_2, t_2^{-1}]$, with $\mathfrak{g}$ the corresponding f.d. simple Lie algebra
   References: [5], [9, Chapter 1,2,7, and 9], [15], [21]
3. **Overview: triangulated categories, derived categories and applications (2 x 45 min).**
   (a) introduction to triangulated and derived categories
   (b) structure of the derived category for hereditary categories
   (c) discussion of certain derived equivalences: coherent sheaves on $\mathbb{P}^1$ and representations of the Kronecker quiver (Beilinson's Theorem), coherent sheaves on a weighted projective line and canonical algebras (tilting theory)
   References: [1], [6], [7]

4. **How to obtain a Lie algebra from a root category: a construction of Peng and Xiao (3 x 60 min).**
   (a) discussion of Kac's Theorem for quivers (correspondence between positive roots of the associated Kac-Moody algebra and dimension vectors of indecomposable representations)
   (b) Is it possible to recover this Lie algebra structure from the data given by the representations? Yes, by the construction of Peng and Xiao.
   (c) root categories, Hall algebra methods and the construction of the Lie algebra

   References: [8], [10], [11], [16], [17]
5. **Overview: Hall algebras (1 x 60 min).**
   (a) Macdonalds ring of symmetric functions and the Hall algebra of nilpotent representations of the Jordan quiver
   (b) Hall algebras of quivers and quantum groups
   (c) Hall algebras of weighted projective lines and quantum loop algebras

   References: [14, Section III.3], [18], [19], [20], [21, Section 2.1, 3.1, 4.6]
6. **Proof of Kac's Theorem for weighted projective lines (4 x 60 min).**
   (a) setup, main theorem, root category and Lie algebra following Peng and Xiao
   (b) map between the loop algebra and the above Lie algebra; state Theorem 2 without proof
   (c) proof of Theorem 1: in detail

   References: [3], [4, Section 6], [10], [12, Section 5]

References

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